LTFAT: The Large Time-Frequency Analysis Toolbox v. 1.4

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Introduction

The Linear Time Frequency Analysis Toolbox (LTFAT) is a Matlab/Octave toolbox for doing time-frequency analysis. It is intended both as an educational and a computational tool.

The goals of the toolbox are:

- To support teaching and learning in Fourier analysis, harmonic analysis and digital signal processing.
- To provide a tested and documentated toolbox of such a qualtive that it can be used for new scientific developments.
- As a method for engineers and researchers to quickly try out a method / transform.
- As a method for researchers to push their discoveries to a larger audience

Key points:

- Started 2003, ongoing development.
- Works in Matlab/Octave with core functions in C
- More than 300 functions
- Matlab GUI for easy coefficient manipulation
- `mat2doc`: A documentation layout system that converts documentation in file headers into HTML and latex files.
- Available from Sourceforge: `http://ltfat.sourceforge.net` with online documentation.

Version 1.3 of the toolbox is a development version intended to integrate and finalize the wavelet module. A the end of the development cycle, it will become Version 2.0

Frame types

Frames are a generalization of bases, allowing for much more flexibility in the construction. LTFAT provides the following bases and frames in an object-oriented framework. All the specific constructions have a fast algorithm for execution of the associated analysis and synthesis transforms.

Description	Frame name
General frames	
Frame specified by matrix	`gen`
Frame specified by operators	`genfun`
Frame with random values	`rand`
Linear frequency scale	
Gabor frame	`dgt`
Gabor frame for real-val. signals	`dgtreal`
Wilson basis	`dwilt`
MDCT	`wmdct`
Logarithmic frequency scale	
Fast Wavelet Transform	`fwt`
Undecimated FWT	`ufwt`
Constant Q-transform	`cqt`
Adaptable frequency scale	
Filterbank	`filterbank`
Uniform filterbank	`ufilterbank`
Filterbank for $\mathbb{R}^L$ only	`filterbankreal`
`ufilterbank` for $\mathbb{R}^L$ only	`ufilterbankreal`
Wavelet tree	`wfbt`
Wavelet packet tree	`wpfbt`
Adaptable time scale	
Non-stationary Gabor transform	`nsdgt`
Uniform nsdgt	`undsgt`
`nsdgt` for $\mathbb{R}^L$ only	`nsdgtreal`
Uniform `nsdgt` for $\mathbb{R}^L$ only	`unsdgtreal`
Pure freqency bases	
Discrete Fourier transform	`dft`
Discrete Cosine transforms	`dcti,dctii,dctiii,dctiv`
Discrete Sine transforms	`dsti,dstii,dstiii,dstiv`
Container frames	
Fusion frame	`fusion`
n-dimensional tensor frame	`tensor`

Frame methods

These are the methods provided by the object oriented framework.

Description	Method name
Creation	
Create a frame	`frame`
Create a frame pair	`framepair`
Create the canonical dual	`framedual`
Create the canonical tight	`frametight`
Accelerate computation	`frameaccel`
Quering a frame	
Frame bounds	`framebounds`
Redundancy	`framered`
Admissible length	`framelength`
Linear operators	
Frame analysis	`frameana`
Frame synthesis	`framesyn`
Matrix representation	`framematrix`
Frame operator diagonal	`framediag`
Coefficient conversions	
Convert to native format	`framecoef2native`
Convert back from native format	`framenative2coef`
Convert to time-frequency layout	`framecoef2tf`
Convert back from TF-layout	`frametf2coef`
Non-linear analysis and synthesis	
Synthesis from magnitude of coefficients	`frsynabs`
Iterative analysis	`franaiter`
Iterative synthesis	`frsyniter`
LASSO threshholding	`franalasso`
Group LASSO thresholing	`franagrouplasso`
Frame multipliers	
Apply frame multiplier	`framemul`
Apply inverse frame multiplier	`iframemul`
Apply the adjoint of a frame multiplier	`framemuladj`
Best approx. by frame multiplier	`framemulappr`
Eigenpairs of a frame multiplier	`framemuleigs`

Fourier analysis

The toolbox features a collection of basic tools from Fourier and time-frequency analysis. These tools are intended mostly for teaching and learning of the mathematical concepts. The emphasis is on documentation and examples. Some highlights:

Fourier analysis:

- Positive frequency Fourier transform for real-valued signals.
- Next efficient FFT size
- Efficient filtering (convolution)
- Discrete Cosine/Sine transforms of type I-IV
- Gaussian, Sech and several classes of Hermite functions
- Classical FIR windows: Sine, Hann, Hamming, Blackman, Nuttall, iterated sine.

Gabor analysis

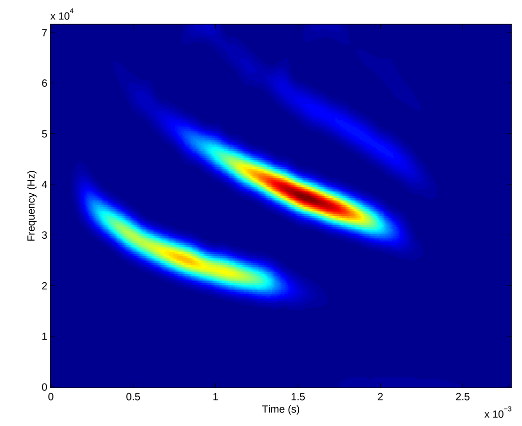
The LTFAT toolbox provides three time-frequency transforms with a equidistant spacing in both time and frequency: The Gabor, Wilson and Modified Discrete Cosine Transform (MDCT).

In addition to the transforms, the toolbox includes twisted convolution, the discrete symplectic Fourier transform, the Zak transform and the S_0-norm.

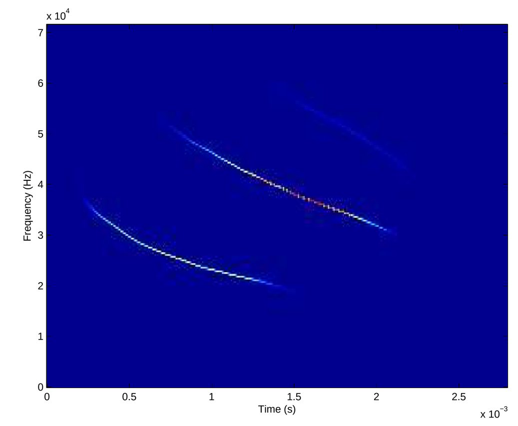
The absolute value squared of the STFT is commonly known as a *spectrogram*. The toolbox contains an easy-to-use plotting routine using the DGT for displaying spectrograms.

The notions of *instantaneous time* and *instantaneous frequency*, also known as *local group delay* are a good way of representing the phase of a short-time Fourier transform.

Reassignment is a method for "deconvolving" a spectrogram. A contribution to a spectrogram at position (x,ω) is in the reassigned spectrogram placed at position $(\text{itime}(x,\omega), \text{ifreq}(x,\omega))$.



Spectrogram of bat chirp



Reassigned bat chirp

Wavelet analysis

The toolbox provides the basic Discrete Wavelet Transform `fwt` and an undecimated version (`ufwt`). The subband channels of these transforms have a logarithmic spacing.

To obtain a more flexible frequency scale, it is possible to construct a wavelet-tree `wfbt` and the wavelet package transform `wpfbt`.

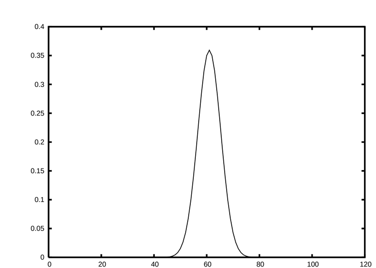
To determine the optimal structure of a wavelet packet tree given an input signal, the best basis algorithm `wpbest` can be used to select the tree structure. This is a non-linear operation.

Besides these transform, the toolbox provides a large number of different wavelet filter types.

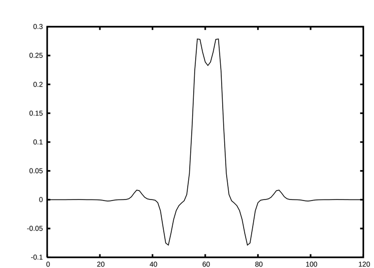
Future plans includes a reassigned wavelet transform similar to the reassigned spectrogram, and an implementation of the Mallat scattering transform.

Windows and filters

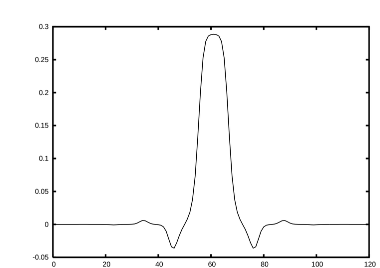
The toolbox can work efficiently with windows that are just as long as the input signals, or with more classic FIR (Finite Impulse Response) windows. For reconstruction, methods for computing the canonical dual and canonical tight window are provided for each transform in frames framework. For wavelets, filters are provided for tight frames / ortonomal bases or in pairs giving perfect reconstruction. The example below shows canonical dual and tight windows for a DGT.



Gauss



Gabdual



Gabtight

Algorithms

The toolbox is build around a few select algorithms with fast implementations in the C programming language. All other routines are then created by pre- and postprocessing algorithms.

- Convolution in the time-domain
- Block-wise convolution by overlap-add
- Block-wise convolution by overlap-save
- Discrete Gabor transformation using an FIR window (weighted overlap-add)
- Discrete Gabor transformation using a full-length window
- The Segmented Discrete Wavelet Transform (SegDWT)

Auditory scales and filters

The toolbox contains a small set of scales and filters, that are commonly used to model the auditory periphery:

- Mel, Bark and Erb scales
- Bandwidth of auditory filters
- Gammatone filters
- μ-law and a-law companding

These functions are used together with the filterbank routines, to create a fully invertible, zero-delay auditory filterbank.

A sister project to LTFAT, the *Auditory Modelling Toolbox* (AMToolbox) aims to collect and improve mathematical models of auditory processing. AMToolbox is build upon LTFAT, and is available on the same terms from `http://amtoolbox.sourceforge.net`.

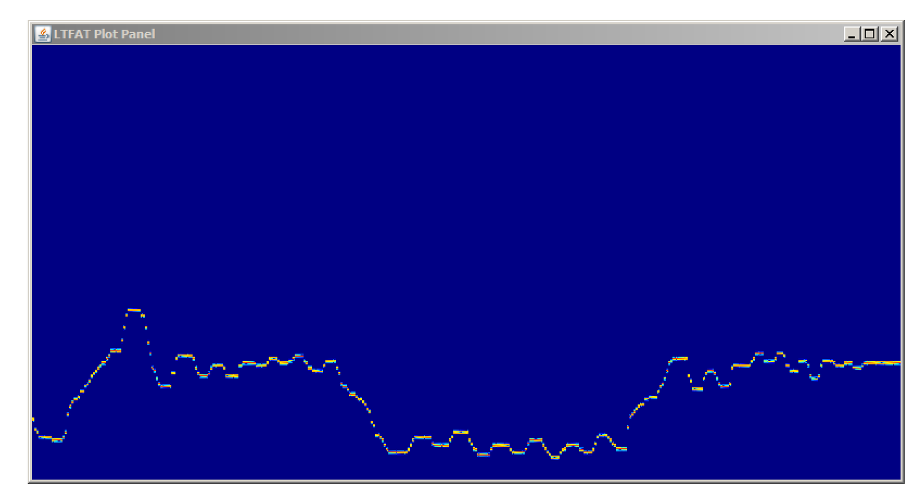
Block processing framework

The block-processing framework allows audio processing in real-time directly from Matlab/Octave.

- Prototype your algorithms in a real-time setting without rewriting the code in a low-level programming language.
- Use time-frequency analysis and synthesis capabilities of the LTFAT to process audio in the transform domain.
- Visualize the coefficients on-the-fly.

Depends on two free, open-source libraries.

- The Portaudio library (`http://www.portaudio.com`): Provides cross-platform audio. A default version distributed with Matlab, but it is replaceable by a custom version possibly supporting additional APIs like Steinberg ASIO SDK or JACK `http://jackaudio.org`
- The Playrec MEX interface (`http://www.playrec.co.uk`): Provides the link from Matlab to Portaudio.



Spectrogram of a single tone after a wild slider movement.

The image shows simple pitch shifting via linear frequency shift of Gabor coefficients. Note: Low-frequency coefficients are zeroed prior to the shifting to avoid shifting the power-line hum when the input is a mic.

References

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